Multiscale Simulation Study of Polymer Systems Based on Dissipative Particle Dynamics

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Multiscale simulations

- Polymer models at coarse-grained (CG) level
- Combining different scales in one simulation
- Enhanced sampling
- Powerful simulation package





Generic polymer model

Dissipative particle dynamics (DPD) :

$$\vec{F}_{i} = \sum_{j \neq i} \left(\vec{F}_{ij}^{c} + \vec{F}_{ij}^{d} + \vec{F}_{ij}^{r} \right)$$

$$\vec{F}_{ij}^{c} = -\nabla V(r_{ij}) = \begin{cases} a_{ij} \left(1 - r_{ij}/r_{c} \right) \hat{r}_{ij} & (r_{ij} < r_{c}) \\ 0 & (r_{ij} \ge r_{c}) \end{cases}$$

$$\vec{F}_{ij}^{d} = -\gamma \omega^{D} \left(r_{ij} \right) \left(\vec{v}_{ij} \cdot \hat{r}_{ij} \right) \hat{r}_{ij}$$

$$\vec{F}_{ij}^{r} = \sigma \omega^{R} \left(r_{ij} \right) \xi_{ij} \hat{r}_{ij}$$

0

$$a\rho = 75 \qquad \qquad \omega^D(r) = \omega^{R^2}(r)$$

$$\chi = 0.2\rho(a_{AB} - a) \qquad \qquad \sigma = (2k_B T \gamma)^{1/2}$$

Generic polymer model

With DPD:



Block copolymers with different sequences, flexibilities, topologies, and so on.



Polymer grafted nanoparticle with different polymer compositions, nanoparticle shapes, and so on.



Solid-state dye-sensitized solar cell

Advantage:

- A large surface/volume ratio of metal oxide
- A short diffusion length for exciton to the interface

Crossland et al., Nano Lett. 9, 2807, 2009.

Gyroid structure only forms in a very narrow composition window (~3%).

Irregular bicontinuous network structure forms in composition window (~10%).



Hillmyer et al. Prog. in Polym. Sci., 33, 875, 2008.

Mahanthappa et al. J. Am. Chem. Soc. 134, 3834, 2012.

nature International weekly journal of science

Materials science: Continuity through dispersity

Richard A Register

Nature **483**, 167–168 (08 March 2012) | doi:10.1038/483167a Published online 07 March 2012



- Whether asymmetric polydispersity is required -that is whether the lengths of A blocks must be narrowly distributed?
- Whether both domains are fully continuous across the entire composition range for which the irregular bicontinuous structure forms?

Schulz-Zimm (SZ) distribution:

$$p(N) = \frac{u^u \delta^{u-1} \exp(-u\delta)}{N_n \Gamma(u)}$$
$$\delta = N/N_n, \text{PDI} = N_w/N_n = (1+u)/u$$

$$A_x B_{N-2x} A_x, N_n = 8^{-18}$$
$$V_{box} = 40 \times 40 \times 40$$

Simulated systems:

System	PDI _A	PDI _B
Asymmetric PDI	1.0	1.5 (SZ)
Symmetric PDI	1.5 (SZ)	1.5 (SZ)

Whether the lengths of A blocks must be narrowly distributed?



Whether the lengths of A blocks must be narrowly distributed?



irregular bicontinuous phase (BIC) ~ 20%

Whether both domains are fully continuous across the entire composition range for which the irregular bicontinuous structure forms?



The BIC structures have good continuity.





Selective distribution of blocks with different chain lengths can stabilize the BIC phase.

Soft Janus particle model



Percec et al. Science, 328, 1009, 2010.

Soft colloidal particle model

We need to describe patchy particles with a simple model. We have proposed a potential to represent the interactions between two colloidal particles:



Soft colloidal particle model

$$U_{ij} = \frac{\alpha_{ij}^R}{2} \left(1 - r_{ij}/r_c\right)^2 - \frac{\alpha_{ij}^A}{2} \left(r_{ij}/r_c - (r_{ij}/r_c)^2\right)$$

$$\alpha_{ij}^R = EV/k_BT$$

$$F_{ij} = -\frac{\partial U}{\partial r} = 0$$

$$\implies d = \frac{\alpha_{ij}^R + \alpha_{ij}^A/2}{\alpha_{ij}^R + \alpha_{ij}^A}$$

d: diameter of colloidal particle

Define $(1+\delta)d = r_c$

 δd : the range of attraction

Substitute r_{ij} by d in U_{ij} , we have $G = \alpha^A_{ij}(1-d)/4$

G: potential well depth

Soft colloidal particle model

$$U_{ij} = \underbrace{\alpha_{ij}^R}_2 (1 - r_{ij}/r_c)^2 - \underbrace{\alpha_{ij}^A}_2 (r_{ij}/r_c) + (r_{ij}/r_c)^2)$$

If we know the modulus of colloid particle (E), the size of the colloid particle (d), the attraction range (δd), and the attraction strength (G), we can exclusively define the parameters in our model.

Suppose:
nanoparticle diameter d = 20 nm $\alpha_{ij}^R \approx 10000$ Elastic modulus $E = 8.3 \times 10^7 \text{ Pa}$ $\alpha_{ij}^A \approx 400$ Attraction range $\delta d = 0.4 \text{ nm}$ $r_c = 1.0 (\approx 20.5 nm)$

Describe the patch size

$$U_{ij} = \frac{\alpha_{ij}^R}{2} \left(1 - r_{ij}/r_c\right)^2 - \underbrace{f^{\nu}}_{2} \frac{\alpha_{ij}^A}{2} \left(r_{ij}/r_c - (r_{ij}/r_c)^2\right)$$

 $f = \cos \theta'_i \cos \theta'_j$ for $|\cos \theta_i| \ge \cos \beta$ and $|\cos \theta_j| \ge \cos \beta$

 $\theta'_i = \arccos(|\cos \theta_i|) \qquad \theta'_j = \arccos(|\cos \theta_j|)$



We then focus on the soft two-patch particle with diameter $d \sim 20$ nm and modulus $E \sim 4.1 \times 10^6$ Pa:





G=9.8 k_BT; β=30°

G=2.0 $k_B T$; β =30°





G=2.0 k_BT; β=60°

G=9.8 k_BT; β=60°

Describe patchy particle



 $f = \begin{cases} \cos \frac{\pi}{2} \left(\frac{\pi/2 - \theta'_i}{\pi/2 - \beta} \right) \cos \frac{\pi}{2} \left(\frac{\pi/2 - \theta'_j}{\pi/2 - \beta} \right) & \text{if } |\cos \theta_i| \le \cos \beta \text{ and } |\cos \theta_j| \le \cos \beta \\ 0 & \text{otherwise.} \end{cases}$

 $\begin{array}{l} \theta_i' = \arccos(|\cos \theta_i|) & \theta_j' = \arccos(|\cos \theta_j|) \\ \beta \in [0, \pi/2] & \end{tabular} \end{tabular} \begin{tabular}{ll} \textbf{The middle part is hydrophobic.} \end{array}$

We then focus on the soft two-patch particle with diameter $d \sim 20$ nm and elastic modulus $E \sim 4.1 \times 10^6$ Pa, and build up phase diagram by scanning the attraction well depth G and the surface coverage β . The volume fraction is $\phi = 0.05$.





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$$U_{ij} = \begin{cases} \frac{\alpha_{ij}^{\kappa} d_{ij}}{2} \left(1 - \frac{r_{ij}}{d_{ij}}\right)^2 - \sum_{\kappa=1}^{M_i} \sum_{\lambda=1}^{M_j} f^{\nu} \left(\mathbf{n}_i^{\kappa}, \mathbf{n}_j^{\lambda}, \mathbf{r}_{ij}\right) \frac{\alpha_{ij}^{\lambda} d_{ij}}{2} \left[\frac{r_{ij}}{d_{ij}} - \left(\frac{r_{ij}}{d_{ij}}\right)^2\right] & r_{ij} \le d_{ij} \\ 0 & r_{ij} > d_{ij}, \end{cases}$$

$$f \left(\mathbf{n}_i^{\kappa}, \mathbf{n}_j^{\lambda}, \mathbf{r}_{ij}\right) = \begin{cases} \cos \frac{\pi \theta_i^{\kappa}}{2\theta_m^{\kappa}} \cos \frac{\pi \theta_j^{\lambda}}{2\theta_m^{\lambda}} & \text{if } \cos \theta_i^{\kappa} \ge \cos \theta_m^{\kappa} \text{ and } \cos \theta_j^{\lambda} \ge \theta_m^{\lambda} \\ 0 & \text{otherwise.} \end{cases}$$

$$(1)$$

$$\begin{aligned} \mathbf{F}_{ij} &= -\frac{\partial U_{ij}}{\partial \mathbf{r}_{ij}} \\ &= \alpha_{ij}^{R} \left(1 - \frac{r_{ij}}{d_{ij}} \right) \frac{\mathbf{r}_{ij}}{r_{ij}} + \sum_{\kappa=1}^{M_{i}} \sum_{\lambda=1}^{M_{j}} \left\{ \alpha_{ij}^{A} f^{\nu} \left(\mathbf{n}_{i}^{\kappa}, \mathbf{n}_{j}^{\lambda}, \mathbf{r}_{ij} \right) \left(\frac{1}{2} - \frac{r_{ij}}{d_{ij}} \right) \frac{\mathbf{r}_{ij}}{r_{ij}} - \frac{\alpha_{ij}^{A}}{2} \left[\frac{r_{ij}}{d_{ij}} - \left(\frac{r_{ij}}{d_{ij}} \right)^{2} \right] \\ &\nu f^{\nu-1} \left(\mathbf{n}_{i}^{\kappa}, \mathbf{n}_{j}^{\lambda}, \mathbf{r}_{ij} \right) \left(\frac{\pi}{2\theta_{m}^{\kappa}} \sin \frac{\pi \theta_{i}^{\kappa}}{2\theta_{m}^{\kappa}} \frac{\partial \theta_{i}^{\kappa}}{\partial \cos \theta_{i}^{\kappa}} \frac{\partial \cos \theta_{i}^{\kappa}}{\partial \mathbf{r}_{ij}} \cos \frac{\pi \theta_{j}^{\lambda}}{2\theta_{m}^{\lambda}} + \frac{\pi}{2\theta_{m}^{\lambda}} \sin \frac{\pi \theta_{j}^{\lambda}}{2\theta_{m}^{\lambda}} \frac{\partial \theta_{j}^{\lambda}}{\partial \cos \theta_{j}^{\lambda}} \\ & \frac{\partial \cos \theta_{j}^{\lambda}}{\partial \mathbf{r}_{ij}} \cos \frac{\pi \theta_{i}^{\kappa}}{2\theta_{m}^{\kappa}} \right) \bigg\}, \end{aligned}$$
(13)
$$\boldsymbol{\tau}_{ij} = \sum_{\nu=1}^{M_{i}} - \frac{\partial U_{ij}}{\partial \mathbf{n}_{i}^{\kappa}} \end{aligned}$$

$$=\sum_{\kappa=1}^{M_i}\sum_{\lambda=1}^{M_j}\frac{\pi\alpha_{ij}^Ad_{ij}}{4\theta_m^\kappa} \left[\frac{r_{ij}}{d_{ij}} - \left(\frac{r_{ij}}{d_{ij}}\right)^2\right]\nu f^{\nu-1}\left(\mathbf{n}_i^\kappa, \mathbf{n}_j^\lambda, \mathbf{r}_{ij}\right)\sin\frac{\pi\theta_i^\kappa}{2\theta_m^\kappa}\frac{\partial\theta_i^\kappa}{\partial\cos\theta_i^\kappa}\cos\frac{\pi\theta_j^\lambda}{2\theta_m^\lambda}\frac{\mathbf{r}_i}{r_{ij}}.$$
 (18)

We use quaternion method to integrate equations of motion.

 $\kappa = 1$







Combining 2 scales in 1 simulation

Reaction models in CG Simulations





Stochastic-reaction-in-a-cutoff method. Can be used to generate polymerization products.

Stochastic reaction in a cutoff



Advantages:

- Simple
- Ready to be implemented in generic/CG models

Stochastic reaction in a cutoff







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Epoxy layer structure on carbon fiber



- Carbon fiber has to be protected by epoxy, otherwise too brittle to use.
- Sizing agent is important to increase the affinity between epoxy layer and carbon fiber.
- In experiments, it's difficult to characterize structures and mechanical properties of this complex.

Chemicals in epoxy and sizing agent



Carbon fiber-epoxy complex



We use DPD to study the influence of reaction on the distribution of different chemicals. The interaction parameters between them are obtained from their χ parameters.

Reaction kinetics

Reaction kinetics:



Reaction+diffusion



Time evolution of epoxy groups



Curing process slows down with time, because the number of functional groups deceases largely with time and "big" molecules are difficult to move.

Chemical distribution

Chemical distribution along the normal direction of carbon fiber surface.



Mechanical properties

- Generate all-atom model based on the chemical distribution in different layers;
- Run MD simulations and calculate mechanical properties.

sample	Up	Middle	Down
	slice	slice	slice
Shear Modulus (GPa)	1.217± 0.244	0.804± 0.461	0.527± 0.385



Timescales (Sampling)

It is still difficult to approach equilibrium even with CG representation.





Integrated tempering sampling (ITS)



Generalized distribution:

 $W(r) = \int_{\beta'} f(\beta') e^{-\beta' U(r)} dr$

$$f(\beta') = \sum_{k} n_k \delta(\beta - \beta')$$

$$W(r) = \sum_{k} n_k e^{-\beta_k U(r)}$$

$$k=1,2,\ldots,N\,.$$

Implementation of ITS

$$e^{-\beta U'(r)} \equiv W(r) = \sum_{k} n_{k} e^{-\beta_{k} U(r)}$$

$$U'(r) = -\frac{1}{\beta} \ln \sum_{k} n_{k} e^{-\beta_{k} U(r)}$$

$$F_{b} = -\frac{\partial U'(r)}{\partial r} = -\frac{\partial U'(r)}{\partial U(r)} \frac{\partial U(r)}{\partial r} = \frac{\sum_{k} n_{k} \beta_{k} e^{-\beta_{k} U(r)}}{\beta \sum_{k} n_{k} e^{-\beta_{k} U(r)}} F$$
For $T_{j} \in [T_{1}, T_{N}]$:

$$\langle A \rangle_{\beta_j} = \frac{\int A(r) e^{-\beta_j U(r)} dr}{\int e^{-\beta_j U(r)} dr} = \frac{\int \frac{A(r) e^{-\beta_j U(r)}}{W(r)} W(r) dr}{\int \frac{e^{-\beta_j U(r)}}{W(r)} W(r) dr} = \frac{\left\langle \frac{A(r) e^{-\beta_j U(r)}}{W(r)} \right\rangle_W}{\left\langle \frac{e^{-\beta_j U(r)}}{W(r)} \right\rangle_W}$$

How to obtain n_k ?

Define the energy U_k^{p} , at which the values of two adjacent terms in W(r) are equal:

$$\begin{split} n_k e^{-\beta_k U_k^p} &= n_{k+1} e^{-\beta_{k+1} U_k^p} \\ \text{Therefore} \\ U_k^p &= \frac{\ln n_k - \ln n_{k+1}}{\beta_k - \beta_{k+1}} \\ \text{For energy } U_{k-1}^p < U < U_k^p \\ n_1 e^{-\beta_1 U} < n_2 e^{-\beta_2 U} < \ldots < n_k e^{-\beta_k U} > \ldots > n_N e^{-\beta_N U} \end{split}$$

The value of *W*(*r*) is dominated by its *k*-th term.

How to obtain n_k ?

Define the energy U_k^{q} , at which the potential energy distribution of the canonical ensemble at temperature T_k is equal to that of the canonical ensemble at temperature T_{k+1} .

$$P_k(U_k^q) = P_{k+1}(U_k^q)$$

Since $P_k(U) = \frac{n(U)e^{-\beta_k U}}{Q_k}$
So $U_k^q = \frac{\ln Q_{k+1} - \ln Q_k}{\beta_k - \beta_{k+1}}$

For energy $U_{k-1}^q < U < U_k^q$

there is a maximum for function $P_k(U)$.



Potential energy

How to obtain n_k ?

To optimize the energy distribution generated in ITS simulation, when W(r) is dominated by the *k*-th term in the range of $U_{k-1}^{p} < U < U_{k}^{p}$, the maximum of the potential energy distribution should be in the same range: $U_{k}^{p} = U_{k}^{q}$

Thus $\ln n_k - \ln n_{k+1} = U_k^q (\beta_k - \beta_{k+1})$

The slope of a secant line is approximated by average of the slopes of tangent lines at two line terminals.

$$U_k^q = \frac{\ln Q_{k+1} - \ln Q_k}{\beta_k - \beta_{k+1}} \approx -\frac{1}{2} \left(\frac{\partial \ln Q_k}{\partial \beta_k} + \frac{\partial \ln Q_{k+1}}{\partial \beta_{k+1}}\right) = \frac{1}{2} \left(\langle U \rangle_k + \langle U \rangle_{k+1}\right)$$

The temperature distribution

Define overlap factor *t*, which gives the ratio between energy distributions at two adjacent temperatures.



$$\beta_k - \beta_{k+1} = \frac{\ln t}{U_k^q - \langle U \rangle_k}$$

The temperature distribution



If a set of temperatures could give a reasonable acceptance ratio in REM simulations, there should be enough overlap between adjacent temperatures in ITS.

Simulation procedure

$$\begin{aligned} \ln n_{k} - \ln n_{k+1} &= U_{k}^{q} (\beta_{k} - \beta_{k+1}) \\ U_{k}^{q} &= \frac{\ln Q_{k+1} - \ln Q_{k}}{\beta_{k} - \beta_{k+1}} \approx -\frac{1}{2} (\frac{\partial \ln Q_{k}}{\partial \beta_{k}} + \frac{\partial \ln Q_{k+1}}{\partial \beta_{k+1}}) = \frac{1}{2} (\langle U \rangle_{k} + \langle U \rangle_{k+1}) \\ \beta_{k} - \beta_{k+1} &= \frac{\ln t}{U_{k}^{q} - \langle U \rangle_{k}} \\ \beta_{k} - \beta_{k+1} &= \frac{\ln t}{U_{k}^{q} - \langle U \rangle_{k}} \\ F_{b} &= -\frac{\partial U'(r)}{\partial r} = -\frac{\partial U'(r)}{\partial U(r)} \frac{\partial U(r)}{\partial r} = \frac{\sum_{k} n_{k} \beta_{k} e^{-\beta_{k} U(r)}}{\beta \sum_{k} n_{k} e^{-\beta_{k} U(r)}} F_{k} \\ 4 \cdot \langle A \rangle_{\beta_{j}} &= \frac{\int A(r) e^{-\beta_{j} U(r)} dr}{\int e^{-\beta_{j} U(r)} dr} = \frac{\int \frac{A(r) e^{-\beta_{j} U(r)}}{W(r)} W(r) dr}{\int \frac{e^{-\beta_{j} U(r)}}{W(r)} W(r) dr} = \frac{\langle \frac{A(r) e^{-\beta_{j} U(r)}}{W(r)} \rangle_{W}}{\langle \frac{e^{-\beta_{j} U(r)}}{W(r)} \rangle_{W}} \end{aligned}$$

5. After ITS simulation, the canonical ensemble properties can be calculated by reweighting.

Coil-to-globule transition



In conventional MD: 31 temperatures

GPU simulation package

GALAMOST: GPU-accelerated large-scale molecular simulation toolkit

http://galamost.com/

Free to download!



Functions:

- CGMD; Brownian Dynamics; Dissipative Particle Dynamics;
- Particle-field coupling (MDSCF);
- Numerical potential (e.g. from iterative Boltzmann inversion & inverse Monte Carlo);
- NVE; NVT; NPT (Nose-Hoover; Andersen);
- Anisotropic soft particle model;
- Stochastic polymerization model;
- Integrated tempering sampling.

GALAMOST: Structures



More characteristics of this package:

Specifically designed for running on GPUs only

Standard format of input and output file: xml, mol2, dcd ...

GALAMOST: Performances

Performances: the average costing time per time step of GALAMOST and HOOMD.

System size: up to 2.2 M LJ liquid particles or 3.0 M DPD liquid particles on GTX 580 with 1.5 GB device memory.



DPD: ~1.5 days for 3.0M particles×1.0M steps.

Summary

Approaching larger spatiotemporal scales in polymer simulations:



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Thank you for your attention!

